

$$E_n = \left(\frac{1}{2} + n\right) \hbar \omega$$

$$P_n = \alpha e^{-\frac{E_n}{k_B T}}$$

$$\alpha e^{-\frac{\hbar \omega}{2k_B T}} \frac{1}{1 - e^{-\frac{\hbar \omega}{k_B T}}} = 1$$

$$\alpha = e^{\frac{\hbar \omega}{2k_B T}} \left(1 - e^{-\frac{\hbar \omega}{k_B T}}\right)$$

$$\begin{aligned} \frac{\hbar \omega}{\left(e^{\frac{\hbar \omega}{k_B T}} - 1\right)^2} &= \hbar \omega \left(-1 - 1\right)^{-1} = -\hbar \omega \left(-1 - 1\right)^{-2} \\ &\quad \cdot \text{since} \\ &= +\hbar \omega \left(-1 - 1\right)^{-2} \cdot e^{\frac{\hbar \omega}{k_B T}} \cdot \frac{\hbar \omega}{k} \frac{1}{T^2} \\ &= \left(-1 - 1\right)^{-2} \frac{(\hbar \omega)^2}{T^2} \frac{1}{k} e^{\frac{\hbar \omega}{k_B T}} \quad \checkmark \end{aligned}$$