$$\frac{9N}{\omega_D^3} \int_0^{\omega_D} \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1} \omega^2 \, d\omega$$

$$x = \frac{\hbar\omega}{kT} \qquad \frac{dx}{d\omega} = \frac{\hbar}{kT}$$

$$d\omega = dx \, \frac{kT}{\hbar}$$

$$\rightarrow \frac{9N}{\omega_D^3} \int_0^{x_D} \frac{kT}{\hbar} \frac{\hbar\omega^3}{e^x - 1} \, dx$$

$$\rightarrow \quad -\frac{1}{4} - \int \frac{x^3}{e^x - 1} \, dx \cdot \left(\frac{kT}{\hbar}\right)^3$$

$$k\Theta_D = \hbar\omega_D$$

$$\rightarrow \left(\frac{k}{\hbar}\right)^3 = \frac{\omega_D^3}{\Theta_D^3} \quad \checkmark$$