Quanten 2 04.04.08

1) $\delta_0 = \frac{\pi}{6}$ $\delta_1 = \frac{\pi}{8}$ $\delta_2 = \frac{\pi}{12}$

Ges: f , $\frac{d\sigma}{d\Omega}$ $P_0 = 1$ $P_2 = \frac{1}{4}(3\cos 2\theta + 1)$

$P_1 = \cos\theta$

$$f_\ell(k) = \frac{e^{2i\delta_\ell(k)} - 1}{2ik} = \frac{1}{k} e^{i\delta_\ell} \sin\delta_\ell$$ (S144)

$$f(k,\theta) = \sum_{\ell=0}^{\infty} (2\ell+1) f_\ell(k) P_\ell(\cos\theta) = \frac{1}{2ik} \sum_{\ell=0}^{\infty} (2\ell+1)\left(e^{2i\delta_\ell} - 1\right) P_\ell(\cos\theta)$$

$$f(k,\theta) = \frac{1}{2ik}\Big[\left(e^{2i\frac{\pi}{6}} - 1\right) + 3\left(e^{2i\frac{\pi}{8}} - 1\right)\cos\theta + 5\left(e^{2i\frac{\pi}{12}} - 1\right) \cdot \frac{1}{4}(3\cos 2\theta + 1)\Big]$$

Plot $\frac{d\sigma}{d\Omega} = |f(k,\theta)|^2$

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b) optisches Theorem

$$\sigma_{tot} = \frac{4\pi}{k} \operatorname{Im}(f(\theta = \emptyset))$$

$$f(\theta = \emptyset) = \frac{1}{2ik}\Big[\left(e^{i\frac{\pi}{3}} - 1\right) + 3\left(e^{i\frac{\pi}{4}} - 1\right) + \frac{5}{\cancel{4}}\left(e^{i\frac{\pi}{6}} - 1\right) \cdot 4\Big]$$

$$= \frac{1}{2ik}\Big[e^{i\frac{\pi}{3}} + 3e^{i\frac{\pi}{4}} + 5e^{i\frac{\pi}{6}} - 9\Big]$$

$$= \frac{i}{2k}\Big[9 - e^{\cdots}\Big]$$

// $e^{i\varphi} = \cos\varphi + i\sin\varphi$

Re $z \cdot e^{i\varphi}$ gibt Im z.

$$\Rightarrow \operatorname{Im} = \frac{1}{2k}\Big[9 - \cos\frac{\pi}{3} - 3\cos\frac{\pi}{4} - 5\cos\frac{\pi}{6}\Big]$$

$$\sigma_{tot} = \frac{4\pi}{k} \cdot \operatorname{Im} = \frac{2\cancel{4}\pi}{\cancel{2}k^2}\Big[9 - \cos\frac{\pi}{3} - 3\cos\frac{\pi}{4} - 5\cos\frac{\pi}{6}\Big]$$

2. Methode: $\sigma_\ell = \frac{4\pi}{k^2}(2\ell+1)\sin^2\delta_\ell$

$$\sigma_{tot} = \frac{4\pi}{k^2}\left[\sin^2\frac{\pi}{6} + 3\sin^2\frac{\pi}{8} + 5\sin^2\frac{\pi}{12}\right]$$

$$\cos 2x = 1 - 2\sin^2 x \longrightarrow \sin^2 x = -\frac{\cos 2x - 1}{2}$$

$$\Longrightarrow \sigma_{tot} = \frac{{}^{2}\cancel{4}\pi}{\cancel{2}k^2}\left[-\cos\frac{\pi}{3} + 1 + 3\left(-\cos\frac{\pi}{4} + 1\right) + 5\left(-\cos\frac{\pi}{6} + 1\right)\right]$$

$$\sigma_{tot} = \frac{2\pi}{k^2}\left[9 - \cos\frac{\pi}{3} - 3\cos\frac{\pi}{4} - 5\cos\frac{\pi}{6}\right]$$