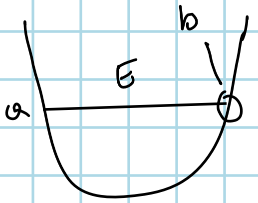
QT 2 VO 28.05.08

WKB weiter:

$$u(x) = \frac{C_+}{\sqrt{p(x)}} e^{\frac{i}{\hbar}\int_{x_0}^{x} dx'\, p(x')} + \frac{C_-}{\sqrt{p(x)}} e^{-\frac{i}{\hbar}\int_{x_0}^{x} dx'\, p(x')}$$

$$p(x) = \sqrt{2m(E - V(x))}$$

umschreiben v. u: $= \frac{c}{\sqrt{p(x)}} \cos\left(\frac{1}{\hbar}\int_{x_0}^{x} dx'\, p(x') + \varphi(x_0)\right)$



$E = V$

$p = \varnothing$

Problem: Das Ganze divergiert

Daher muss man annäh. treffen

Näherung: $V(x) \approx \underbrace{V(b)}_{E} + V' \cdot (x - b)$

Lsg d SGL:

$$-\frac{\hbar^2}{2m} u'' + V(x)\, u(x) = E\, u(x)$$

$$u'' = \frac{2m}{\hbar^2}(V - E)\, u \approx \underbrace{\frac{2m V'(b)}{\hbar^2}}_{c} (x - b)\, u$$

$$(u'' - z u = \varnothing)$$

$$\frac{d^2 u}{dx^2} - c(x - b)\, u = \varnothing \qquad // \; z = c_b (x - b)$$

$$// \; \frac{d^2 u}{dx^2} = c_b^2 \frac{d^2 u}{dz^2} \qquad // \text{ Koord - Trafo}$$

$$c_b^2 \frac{d^2 u}{dz^2} - \frac{c}{c_b} z u = \varnothing$$

$$u'' - \frac{c}{c_b^3} z u = \varnothing \qquad // \text{ setze } \frac{c}{c_b^3} = 1$$

$B_i(x)$: - - -   siehe Folie

$A_i(x)$ ——   Lsg: Airy Funkt.

B_i divergent

$\Rightarrow A_i$ physik. Lösung

$$A_i(-z) \xrightarrow[z\to\infty]{} \frac{\cos\left(\frac{2}{3} z^{\frac{3}{2}} - \frac{1}{4}\pi\right)}{2\sqrt{\pi}\, z^{\frac{1}{4}}}$$

$$A_i(z) \xrightarrow[z\to\infty]{} \frac{\exp\left(\overbrace{-\frac{2}{3} z^{\frac{3}{2}}}^{\text{wichtig!}}\right)}{\sqrt{\pi}\, z^{\frac{1}{4}}}$$

WKB Lsg in der Nähe v. b

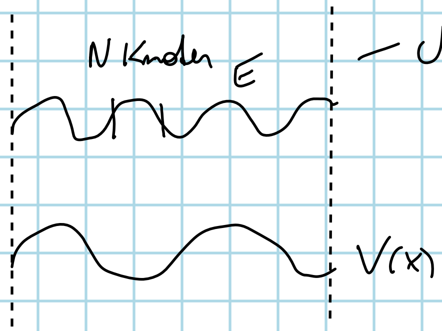
$$u_b(x) = A_i\left(c_b(x-b)\right)$$

$$u_a(x) = A_i\left(C_a(a-x)\right)$$

$$u_a^{WKB} = C \frac{\cos\left(\frac{1}{\hbar}\frac{2}{3}\sqrt{-2mV'(a)}\,(x-a)^{\frac{3}{2}} - \varphi_a\right)}{\sqrt[4]{(a-x)\,2mV'(a)}}$$

$$= C \frac{\cos\left(\frac{2}{3} z^{\frac{3}{2}} + \varphi_a\right)}{z^{\frac{1}{4}}} \qquad \boxed{\varphi_a = -\frac{\pi}{4}}$$

Airy Fkt. kann nicht Phase = ∅ haben



— Neumann RB -- Ableitg am Rand gegeben

$$u^{WKB} = \frac{C}{\sqrt{p(x)}} \cos\left[\frac{i}{\hbar}\int dx'\, p(x') + \cancel{\varphi_A}\right]$$

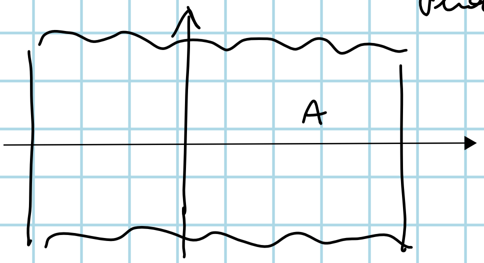
streichen wegen Neumann?

N Knoten: $\frac{1}{\hbar}\int_a^b dx'\, p_E(x') := N\pi$

Umschreiben $\int$ in andere Richt. p.

$$\underbrace{\left[\int_a^b p(x) + \int_b^a -p(x)\right]}_{A} = 2\pi\hbar N$$

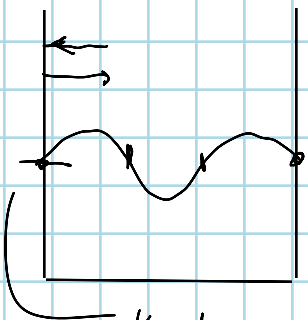
Phasenraum



für Neumann RB exakt.

$$\boxed{A = \oint p\, dq = 2\pi\hbar\, n}$$

Dirichlet RB:



Kanten am Rand, klassisch eine Reflexion

p wird 0 und klappt um! $p \to -p$

$$\frac{1}{\sqrt{p}} \to \frac{1}{\sqrt{-p}} = \frac{\pm i}{\sqrt{|p|}} = \frac{e^{\pm i\frac{\pi}{2}}}{\sqrt{|p|}}$$

Phasen: Neumann: 0

Dirichlet: $-\frac{\pi}{2}$

Weiche (Airy Fkt): $-\frac{\pi}{4}$

A korrigiert: $\boxed{\oint p\, dq = 2\pi\hbar\left(n + \frac{N}{4}\right)}$

<u>Maslov Index</u> $N := 0 + N_{\text{weich}} + 2N_{\text{Hard}}$