Statistik Test Vorber.

Test SS 07 12. 10. 07

1) $H = \sum_{i=1}^{3N} \left(\frac{p_i^2}{2m} + \frac{1}{2} m\omega^2 q_i^2 \right) = \sum_i^N \hat{p}^2 + \hat{q}^2$

$\Omega = \frac{d\phi}{dE} \cdot \Delta E$ $\qquad \phi = \int d^{3N}q \, d^{3N}p =$

$\hat{p} = \frac{p}{\sqrt{2m}}$

$\hat{q} = q\sqrt{\frac{m\omega^2}{2}}$

$dp = d\hat{p}\,\sqrt{2m}$

$dq = d\hat{p}\,\sqrt{\frac{2}{m\omega^2}}$

$\rightarrow \phi = (\sqrt{2m})^{3N} \left(\sqrt{\frac{2}{m\omega^2}}\right)^{3N} \int d^{3N}\hat{q}\, d^{3N}\hat{p}$

$= \left(\frac{2}{\omega}\right)^{3N} \cdot V_{Kugel}^{6N}$ $\qquad R = \sqrt{E}$

$= \left(\frac{2}{\omega}\right)^{3N} \frac{(\cancel{2}\pi E)^{3N}}{\cancel{2^{3N}}\,(3N)!}$ ✓

$\Omega = \underbrace{\frac{1}{h^{3N} N!}}_{c} \frac{d\phi}{dE} \Delta E = \frac{1}{h^{3N} N!} \frac{1 \cdot 3N \cdot E^{3N-1}}{\omega^{3N}\, 3N!} \cdot \Delta$

$= \underbrace{\frac{1}{h^{3N} N!\, \omega^{3N}} \frac{E^{3N-1}}{(3N-1)!}}_{C} \cdot \Delta$

b) $C_V = \left(\frac{\partial E}{\partial T}\right)_{V,N}$

S E V

H F

P G T

$S = k_B \ln \Omega$

$\frac{\partial E}{\partial S} = T$ eq. $\frac{\partial S}{\partial E} = \frac{1}{T}$

$S = k_B \ln\left(\frac{\Delta}{C} E^{3N-1}\right) = k_B\left(\ln\frac{\Delta}{C} + \ln E^{3N-1}\right)$

$\frac{\partial S}{\partial E} = \frac{\partial}{\partial E} k_B(\ln E^{3N} - \ln E) = \frac{\partial}{\partial E} k_B(3N-1)\ln E$

$\Rightarrow \frac{1}{T} = \frac{k_B(3N-1)}{E}$ $\qquad E = (3N-1)k_B T$

$\frac{\partial E}{\partial T} = (3N-1)k_B = C_V$

Test 14.5.07

$S(E,V) = A E^{\frac{1}{2}} + B V^{\frac{1}{2}}$

$$\begin{array}{ccc} S & E & V \\ H & & F \\ p & G & T \end{array}$$

a) Berechne p & $E(V,T)$

$dE = T dS - p dV$

$\rightarrow \quad dS = \frac{dE}{T} + \frac{p}{T} dV$

$dS = \frac{\partial S}{\partial E} dE + \frac{\partial S}{\partial V} dV = \frac{A}{2 E^{\frac{1}{2}}} dE + \frac{B}{2 V^{\frac{1}{2}}} dV$

$\Rightarrow \quad \frac{1}{T} = \frac{A}{2 E^{\frac{1}{2}}} \quad \& \quad \frac{p}{T} = \frac{B}{2 V^{\frac{1}{2}}}$

$\left(\frac{A T}{2}\right)^2 = E \qquad p = \frac{B T}{2 V^{\frac{1}{2}}}$

b) i $\quad dE = \delta Q + \underset{-p\,dV}{\delta W}$ $\qquad p_1 \rightarrow p_2, \quad V_1 \rightarrow V_2$

$\Rightarrow dQ = dE + p\,dV$

$Q = \int_{p_1}^{p_2} dQ + \underbrace{\int_{V_1}^{V_2} dQ}_{\text{hier } dp = 0}$

hier $dV = 0$

$$\boxed{\begin{aligned} E &= \frac{A^2 T^2}{4} \\ p &= \frac{B T}{2\sqrt{V}} \rightarrow T = \frac{p\sqrt{V}\,2}{B} \\ \Rightarrow E &= \frac{A^2 p^2 V \cancel{4}}{\cancel{4} B^2} \\ dE &= \frac{A^2}{B^2}(2pV\,dp + p^2\,dV) \end{aligned}}$$

$dQ = \frac{A^2}{B^2}(2pV\,dp + p^2\,dV) + p\,dV$

$\rightarrow Q = \frac{A^2}{B^2}\int_{p_1}^{p_2} 2p\,V_1\,dp + \int_{V_1}^{V_2} \frac{A^2}{B^2} p_2^2 + p_2\,dV$

$= \frac{A^2}{B^2} V_1 (p_2^2 - p_1^2) + \left(\frac{A^2}{B^2} p_2^2 + p_2\right)(V_2 - V_1)$

$$c = \frac{\partial Q}{\partial T} \qquad\qquad đQ = dE - đW$$
$$= dE + p\, dV$$

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V \qquad // \qquad E = \frac{A^2 T^2}{4}$$

$$\Rightarrow \quad c_v = \frac{A^2 T}{2} \qquad\qquad V = \frac{B^2 T^2}{p^2 4}$$

$$C_p = \left(\frac{\partial Q}{\partial T}\right)_p = \frac{\partial E}{\partial T} + p \frac{\partial V}{\partial T} = \frac{A^2 T}{2} + \cancel{p} \frac{B^2 \cancel{2} T}{p^{\cancel{2}} \cancel{4} 2}$$
$$= \frac{A^2 T}{2} + \frac{B^2 T}{2}$$

ii) $(p_1, V_1) \rightarrow (p_2, V_2)$

$$x = \frac{p}{V} \qquad\qquad dV = \frac{dp}{x}$$
$$V = \frac{p}{x}$$

$$đQ = \frac{A^2}{B^2}\left(2pV\, dp + p^2\, dV\right)$$
$$+ p\, dV = \frac{A^2}{B^2}\left(\frac{2p^2}{x} dp + \frac{p^2}{x} dp\right)$$
$$+ \frac{p}{x} dp = \frac{A^2}{B^2}\left(\frac{3p^2}{x} dp\right)$$
$$+ \frac{p}{x} dp$$

$$Q = \int_{p_1}^{p_2} \frac{A^2}{B^2}\frac{3}{x} p^2\, dp + \frac{p}{x} dp$$
$$= \frac{A^2}{B^2 x}\left(p_2^3 - p_1^3\right) + \frac{\left(p_2^2 - p_1^2\right)}{2x} \quad ✓$$

$$x = pV \qquad\qquad x = p_1 V_1$$

$$\langle A \rangle = \langle \alpha | A | \alpha \rangle$$

$$\langle \uparrow | S_x | \uparrow \rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} =$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} \quad \underline{\underline{0}}$$