(5)

Gas mit N Teilchen Vol V Wärmebad T

$$H = \sum_{i=1}^{N} \frac{\vec{p}_i^{\,2}}{2m}$$

$$Z_k = \frac{1}{N! h^{3N}} \int e^{-\beta \sum_i \frac{\vec{p}_i^{\,2}}{2m}} d^{3N}q \, d^{3N}p$$

$$= \frac{V^N}{N! h^{3N}} \int e^{-\beta \sum_i \frac{\vec{p}_i^{\,2}}{2m}} d^{3N}p$$

$$= C_1 \prod_i^{3N} \int e^{-\beta \frac{p_i^2}{2m}} dp_i \qquad \tilde{p} = \sqrt{\frac{\beta}{2m}}\, p$$

$$dp = \sqrt{\frac{2m}{\beta}}\, d\tilde{p}$$

$$= C_1 \left(\sqrt{\frac{2m}{\beta}}\right)^{3N} \underbrace{\prod_i^{3N} \int e^{-\tilde{p}^2} d\tilde{p}_i}_{\left(\sqrt{\pi}\right)^{3N}} \quad \checkmark$$

$$\frac{V^N}{N! h^{3N}}$$

$$F = -kT \ln Z_k = -\beta^{-1} \ln \underbrace{\frac{V^N}{N! h^{3N}} \left(\sqrt{\frac{2m}{\beta}}\right)^{3N} \left(\sqrt{\pi}\right)^{3N}}_{\frac{V^N}{N!}\left(\sqrt{\frac{2m\pi}{\beta h}}\right)^{3N}} \Big\} Z_K$$

$$= \frac{1}{\beta} \ln \frac{N!}{V^N} \left(\sqrt{\frac{\beta h}{2m\pi}}\right)^{3N}$$

b) $\langle E \rangle = -\partial_\beta \ln Z_k$

$$= -\frac{1}{Z_k} \partial_\beta Z_k =$$

$$+\frac{N!}{V^N} \left(\frac{\beta h}{2m\pi}\right)^{\frac{3N}{2}} \frac{V^N}{N!} \frac{3N}{2} \left(\frac{2m\pi}{\beta h}\right)^{\frac{3N}{2}-\frac{2}{2}} \frac{2m\pi}{h\beta} \frac{1}{\beta}$$

$$= \frac{3N}{2\beta} = \frac{3}{2} N k_B T \quad \checkmark$$

$$\langle S \rangle = -k_B \langle \ln \rho_k \rangle \qquad \rho_k = \frac{1}{Z_k} e^{-\beta H}$$

$$\ln \frac{1}{Z_k} + \ln e^{-\beta H} = -\beta H - \ln Z_k$$

$$\langle S \rangle = k_B \left(\langle \beta H \rangle + \langle \ln Z_k \rangle \right) \qquad // \langle A \rangle = \mathrm{Tr}(\rho A)$$

$$= k_B \left(\underbrace{\frac{1}{N! \ell^{3N}} \int \frac{1}{Z_k} e^{-\beta H} \beta H}_{\beta \langle E \rangle} + \underbrace{\frac{1}{N! \ell^{3N}} \int \frac{1}{Z_k} e^{-\beta H}}_{1 \text{ norm...}} \ln Z_k \right)$$

$$\langle S \rangle = k_B \left(\ln Z_k + \beta \langle E \rangle \right)$$

$$F = -\frac{\ln Z_k}{\beta}$$

$$H = \sum_i^N -\mu_i B$$

2) $$Z_k = \sum_{\{\mu_1, \mu_N\}} e^{+\beta \sum_i^N \mu_i B}$$

$$= \sum_{\mu_1} \sum_{\mu_2} \sum \cdots e^{\beta(\mu_1 B + \mu_2 B + \cdots)}$$

$$= \cdots e^{\beta \mu_1 B} \cdot e^{\beta \mu_2 B} \cdot \cdots$$

$$e^{\beta +} \quad e^{\beta \mu_2} \quad - - - \quad + \quad e^{\beta -} \quad c^{\wedge 2}$$

$$\langle M \rangle = \left\langle \sum_i^N \mu_i \right\rangle = \mathrm{Tr}\left(\rho_k \sum_i^N \mu_i \right)$$

$$\boxed{\rho_k = \frac{1}{Z_k} e^{-\beta H}}$$

$$= \mathrm{Tr}\left(\frac{1}{Z_k} e^{-\beta H} \sum_i^N \mu_i \right)$$

$$= \sum_{\{\mu_1, \mu_N\}} \frac{1}{Z_k} \underbrace{e^{-\beta H} \sum_i^N \mu_i}_{\frac{1}{B} \frac{\partial}{\partial \beta} e^{-\beta H}}$$

$$H = -\sum_i \mu_i B$$

$$= \sum_{\{\mu_1,\mu_N\}} \frac{1}{Z_k} \frac{1}{B} \partial_\beta e^{-\beta H} = \frac{1}{Z_k} \frac{1}{B} \partial_\beta \underbrace{\sum_{\{\mu\}} e^{-\beta H}}_{Z_k}$$

$$\rightarrow \quad M = \frac{1}{Z_k} \frac{1}{B} \partial_\beta Z_k$$

Z_k einsetzen – fertig

(6) 1) idealen Gas im Falle

$$H = \frac{\vec{p}^2}{2m} + \frac{1}{2} m\omega^2 \vec{x}^2$$

$$Z_{Gk} = \sum_{N=0}^{\infty} \frac{1}{N! \, h^{3N}} \int e^{-\beta(H-\mu N)} \, d^{3N}q \, d^{3N}p$$

$$z = e^{\beta\mu} \quad \rightarrow \quad \sum C_k \int e^{-\beta H} z^N \, d^{3N}q \, d^{3N}p$$

$$= \sum C_k \prod_i^{3N} \int e^{-\beta \frac{p_i^2}{2m}} dp_i \prod_j^{3N} \int e^{-\beta \frac{m\omega^2 x_i^2}{2}} dq_i$$

$$\tilde{p} = \sqrt{\frac{\beta}{2m}} \, p \qquad \tilde{x} = \sqrt{\frac{\beta m\omega^2}{2}} \, x$$

$$dp = \sqrt{\frac{2m}{\beta}} \, d\tilde{p} \qquad dx = \sqrt{\frac{2}{\beta m\omega^2}} \, d\tilde{x}$$

$$Z_{Gk} = \sum_{N=0}^{\infty} \frac{1}{N! \, h^{3N}} \left(\sqrt{\frac{2m}{\beta}}\right)^{3N} \left(\sqrt{\frac{2}{\beta m\omega^2}}\right)^{3N} \pi^{3N} z^N$$

$$= \sum_N \frac{1}{N!} \left(\frac{1}{\hbar\beta\omega}\right)^{3N} z^N = e^{\frac{z}{(\beta\hbar\omega)^3}} \quad \checkmark$$

$$Z_{Gk} = \sum_N z^N \cdot Z_k = \sum_N e^{\beta\mu N} Z_k$$

$$\langle N \rangle = \frac{1}{\beta} \partial_\mu \ln Z_{GK}$$

ln weil Z_{GK} eine e-Fkt. ist!

$$= \frac{1}{\beta} \partial_\mu \frac{e^{\beta\mu}}{(1+e^{\beta\mu})^3} = \frac{2}{(1+\beta\mu)^3}$$